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LETTER TO THE EDITOR

On the physical interpretation of a transitional amplitude in percolation theory

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Abstract. This letter deals with the physical interpretation of the 'transitional amplitude' ΔE, a parameter of percolation theory determined by the derivative mean size of clusters $\bar{S}$: $\Delta E = x(\mathrm{d}\bar{S}/\mathrm{d}x) + \bar{S}$; $\Delta E = \bar{S}(1 + L_{\mathrm{R}}) = \varepsilon^{-(\gamma+1)}$, where $\varepsilon = x - x_{\mathrm{c}}$, and L_{R} is the average number of red bonds in an incipient infinite cluster.

When applying percolation theory in some of the branches of physics, particularly in the physics of fracture, there arises a problem of description of the transient dynamic reaction of the system to the unit action. Consider, for example, the well known empirical law for dependence of frequency of occurrence of acoustic pulses registered during fracture of solids, N, on their amplitude M,

$$\log N(M) = a - bM, \tag{1}$$

where a and b are constants for a given sample. Law (1) holds at various scales of fracture—from acoustic emission (AE) of small samples to earthquakes. Naturally, it is very important to derive this law from 'first principles'.

An attempt to derive (1) on the basis of the theory of stochastic processes was made by Otsuka (1972), Vere-Jones (1977), Murty (1983) and others. In their models the acoustic pulse amplitude M is correlated with the cluster size of elementary defects.

In this paper we present a model where A is assumed to correspond to an increment of the size of merging clusters of defects due to the addition of a single defect. The AE amplitude problem is part of the percolation model of fracture suggested by Chelidze (1982, 1983) and Chelidze and Kolesnikov (1982, 1983).

Let us consider the traditional percolation problem which is the clustering of occupied sites with an increase of their concentration x (Stauffer 1979), assuming that the occupied site is the damaged one. x will be understood hereafter as the probability for a given site to be damaged.

If it is assumed that the addition of each new occupied site to the system will produce a certain dynamic response—a transitional process of the type of acoustic emission, electromagnetic radiation, etc—then it should be assumed that the amplitude of this response depends on the state of neighbouring sites.

Clearly, the transitional amplitude should increase with an increase in the number of large clusters, i.e. with an increase of x, as suggested by Otsuka, Vere-Jones and Murty.

However, at the same time it should be taken into account that formation of a large cluster is not a single-act process and, moreover, the bulk of clusters remains unchanged in the case of a single percolation act: only a minority of clusters become involved in the transitional process caused by the addition of a single site and this is overlooked by the cited authors.

To take this phenomenon into consideration, Chelidze and Kolesnikov (1983) suggested another model of acoustic pulse generation. Specifically, the effective amplitude A which is a function of size of merging clusters was introduced:

$$A = A_0\left\{\left[\left(\sum_{i=1}^{k+1} s_i\right)^2 - \sum_{i=1}^{k} s_i^2\right]^{1/2}\right\}, \tag{2}$$

where k is the number of clusters merging with an added site, s_i is the number of sites in merging clusters, and A_0 is an effective amplitude generated at occupation of a single isolated site.

Our aim is to show that the empirically introduced function A, which it is better to call the transitional amplitude, can be correlated with the fundamental functions of percolation theory.

To calculate the transitional amplitude, the following model is adopted. Isolated sites forming the so-called single cluster will be described by a transitional amplitude A_0.

To estimate the system's reaction to the merging of clusters for $s \geqslant 1$, it is reasonable to assume that the transitional amplitude is the larger, the greater an increase of the cluster size caused by the addition of a new single site.

In terms of physics, this can be understood as the dependence of A on the change of energetic state of sites and this change is proportional to an increment in the cluster size.

As is known, the mean size of a finite cluster is determined by

$$\bar{S}(x) = \sum_s s^2 n_s(x) \Big/ \sum_s s n_s(x). \tag{3}$$

On differentiating (3) with respect to x and taking into account that for $x < x_c$ the sum $\Sigma_s\, s n_s(x) = x$, we get

$$\frac{\mathrm{d}}{\mathrm{d}x} \sum_s s^2 n_s(x) = x \frac{\mathrm{d}\bar{S}(x)}{\mathrm{d}x} + \bar{S}(x). \tag{4}$$

Let us consider the increment of the sum in the left-hand part of this equation due to the addition of a single occupied site to the system. Designating the number of clusters which will merge with the added one by k (k can vary from 0 to z, where z is the coordination number), the sum in (4) before the addition of a new site can be rewritten as

$$\sum_s s^2 n_s(x) = \sum_j^K s_j^2 + \sum_i^k s_i^2 \tag{5}$$

where i and j are summation indexes over k clusters which will participate in the act of merging and over K clusters which will not participate in it, respectively.

Then, after the addition of a new occupied site (the latter is considered as a unit cluster), the sum (5) is designated by Σ_s^*. It can be written down as

$$\sum_s^* s^2 n_s(x) = \sum_j^K s_j^2 + \left(\sum_i^{k+1} s_i\right)^2 \tag{6}$$

because after the merging of k clusters of sizes s_i with a new site there appears a new cluster of size $\Sigma_i^{k+1} s_i$.

Inserting in (4) the difference of (6) and (5), equating it with the initial expression for the transitional amplitude (2) and denoting $(\langle A\rangle/A_0)^2$, where $\langle A\rangle$ is the mean of A-values obtained after many repetitions, by ΔE, we get

$$\Delta E = \left\langle\left[\left(\sum_i^{k+1} s_i\right)^2 - \sum_i^k s_i^2\right]\right\rangle = x\frac{\mathrm{d}\bar{S}}{\mathrm{d}x} + \bar{S}(x). \tag{7}$$

Obviously, the value ΔE, which is expressed by the fundamental percolation function $\bar{S}(x)$ and its derivative with respect to concentration, gives new information compared with $\bar{S}$. If when calculating sizes of clusters $\bar{S}$ by formula (2) the mode of their formation is ignored (statistics takes into account only the finite cluster size), the value ΔE depends essentially on a cluster formation mode. Naturally ΔE increases with the decrease of $x - x_c = \varepsilon$, since the probability of merging of large clusters also increases and for $\varepsilon \to 0$ we have

$$\Delta E \sim \varepsilon^{-(\gamma+1)}.$$

It is interesting to note that the value ΔE can be expressed by the average number of red (cutting) bonds in an incipient infinite cluster L_R (Coniglio 1982),

$$L_R = (x/\bar{S})\,\mathrm{d}\bar{S}/\mathrm{d}x \sim \varepsilon^{-1}.$$

Therefore $\Delta E = \bar{S}(1 + L_R) = \varepsilon^{-(\gamma+1)}$.

As comparison with experiment shows, the function $(\Delta E)^{1/2}$ describes well the distribution of amplitudes of elastic pulses generated at sample destruction. Specifically, the quantity $\gamma' = 1/(\gamma+1)$ is equal to ~ 0.3 for a two-dimensional and to ~ 0.4 for a three-dimensional system and these values are in accordance with the experimentally obtained values of factor γ' in the empirical equation $\gamma' = \Delta \log N_\varepsilon / \Delta \log E'$, where E' is the energy of acoustic pulses ($\log E \sim M$) and N_ε is the cumulative number of pulses with an amplitude $M \geqslant M_i$.

It is possible that ΔE reflects different kinds of energy losses at clustering due to heating, acoustic emission, electromagnetic and other types of radiation, i.e. it is the measure of energy losses at each act of percolation.

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